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Some Comments on the Modeling of the Turbulent Wake of a Self-Propelled Body in a Stratified Fluid

by

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Abstract

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Model Scaling

In this note, the modeling of the turbulent wake of a self-propelled body of length L traveling at a speed U in a stratified fluid which is characterized by a Vaisala frequency N is discussed. The diameter of the body is D, the diameter of the propeller is d and it will be assumed that $L/D = 10$ and $D/d = 2$ so that $L/d = 20$. For the full scale or prototype body, typical numerical values are U = 6 knots $\approx$ 3 m/sec, $L = 100\ m$ and $N = 10^{-2}\mathrm{sec}^{-1}$.

The scaling parameter is the Froude number

$$F = U/(ND)$$

which is, of course, equivalent to Richardson number scaling since

$$Ri = 1/F^2 .$$

In the usual way, U, D and N are chosen so that

$$F_P = \left(\frac{U}{ND}\right)_P = \left(\frac{U}{ND}\right)_M = F_M,$$

where the subscripts P and M refer to the values of the parameters for the prototype and model respectively.

In both the model and prototype, the physics must be the same for a meaningful model experiment. The propeller produces turbulence. The wake grows by entraining fluid with a consequent decrease in the turbulent energy and increase in the potential energy of the fluid within the wake. Simultaneously there is viscous dissipation of the turbulence energy. Finally, when the turbulent energy drops to a sufficiently low level, the wake collapses.

One of the consequences of Ko's model of the growing and collapsing wake (Ko, 1971) is that the time for the wake to collapse, t_c, is nearly independent of the Froude number and is

$$t_c \approx 2/N.$$

This is, of course, in agreement with the available experimental data (Schooley and Stewart, 1963; Van der Watering, 1966). Another important time scale is that of the turbulence (Batchelor, 1967)

$$t_T = \ell/u$$

where ℓ is the integral scale of the turbulence and u is the turbulent intensity.

Clearly, the ratio between these times must be equal for both model and prototype if the phenomenon of wake growth and collapse is to be correctly modeled. This ratio is

$$t_c/t_T = 2u/(N\ell)$$

The turbulent intensity is a fraction of U,

$$u = \alpha U \quad , \quad 0 < \alpha < 1.$$

Ko suggests that $\alpha \approx 0.25$ just behind the propeller and this is in rough agreement with the results obtained by Naudascher (1965). It is also to be expected that the integral scale is a fraction of the propeller diameter, d, i.e.,

$$\ell = \beta d \quad , \quad 0 < \beta < 1.$$

With these assumptions

$$\left(\frac{t_c}{t_T}\right)_M = \left(\frac{2u}{N\ell}\right)_M = 2\left(\frac{\alpha U}{\beta N d}\right)_M$$

$$= 4\left(\frac{\alpha}{\beta}\right)_M \left(\frac{U}{ND}\right)_M = 4\left(\frac{\alpha}{\beta}\right)_M F_M,$$

and

$$\left(\frac{t_c}{t_T}\right)_P = \left(\frac{2u}{N\ell}\right)_P = 2\left(\frac{\alpha U}{\beta N d}\right)_P$$

$$= 4\left(\frac{\alpha}{\beta}\right)_P \left(\frac{U}{ND}\right)_P = 4\left(\frac{\alpha}{\beta}\right)_P F_P$$

Therefore

$$\left(\frac{t_c}{t_T}\right)_M = \left(\frac{t_c}{t_T}\right)_P$$

if, and only if

$$\left(\frac{\alpha}{\beta}\right)_M = \left(\frac{\alpha}{\beta}\right)_P .$$

That is, the ratio of the time for the wake to collapse to the characteristic time of the turbulence will have the same value for both the model and the prototype if the turbulent intensity is the same fraction of the body speed and the integral scale is the same fraction of the propeller diameter in both model and prototype.

In both model and prototype the self-propulsion is accomplished by a propeller; both the model and prototype propellers are geometrically similar and the ratio of the rotational speed of the propeller to body speed is the same

for both model and prototype. Since both the geometry and the velocity triangles of the blades are similar, it is expected, provided that the model propeller produces a turbulent flow, that the ratios of the integral scale to the diameter and the turbulent intensity to the body speed are invariant with scale. That is, it can be expected that α and β are truly constants. Then the ratio (t_c/t_T) is also invariant. Finally, this ratio can be estimated if it is assumed that $\alpha = 1/4$ and $\beta \approx 1/2$,

$$\left(\frac{t_c}{t_T}\right) = 2\left(\frac{U}{ND}\right)_p \approx 100.$$

These estimates suggest that <u>the collapsing wake can be properly modeled at any scale provided that the wake is turbulent at that scale.</u> But, is the wake turbulent behind the propeller and, say, up to the point of collapse? It is clear that the Reynolds number will be very different in model and prototype, but there must be a similarity in that a buoyancy range and an inertial range must be present in the turbulent spectrum at model scale if they are present at prototype scale.

Therefore consider the conditions at two sections; just behind the propeller and at the point of collapse. Taking the time of collapse as

$$t_c = 2/N,$$

the point of collapse is

$$x_c = Ut_c = 2U/N.$$

Therefore

$$\left(\frac{x_c}{d}\right) = 2\left(\frac{U}{Nd}\right) = 2\left(\frac{D}{d}\right)\left(\frac{U}{ND}\right) \approx 120$$

using the values of U, D and N for the prototype. So the state of the flow must be estimated at

$$x = 0 \qquad \text{and} \qquad x = 120\,d$$

In all the numerical work below, it will be assumed that the speed of the model, U_M = 1 m/sec.

<u>Conditions for the Existence of a Turbulent Wake</u>

A necessary condition for the existence of an inertial range in the turbulent spectrum (Phillips, 1966) is that

$$R_\ell^{1/2} = \left(\frac{u\ell}{\nu}\right)^{1/2} \gg 1.$$

It will be assumed that, just behind the propeller (x = 0),

$$u = U/4 \quad , \quad \ell = d/2,$$

then

$$R_{\ell_0}^{1/2} = \left(\frac{u\ell}{\nu}\right)^{1/2} = \left(\frac{Ud}{8\nu}\right)^{1/2}\left(\frac{d}{L}\right)^{1/2}.$$

With U = 1 m/sec, d/L = 1/20, and ν = 10^{-2} cm^2/sec,

$$R_{\ell_0}^{1/2} \approx 8\,L^{1/2}$$

with L in cm. It is clear that any model longer than about 50 cm easily satisfies the condition $R_{\ell_0}^{1/2} >> 1$.

Next, estimate $R_\ell^{1/2}$ at $x = 120d$. Estimates of u and ℓ at $x = 120d$ are required. The only experimental results which are available, those of Naudascher (1965), will be used. Using the data in Naudascher's Table 3 and Figure 15*

$$\ell/d \approx 0.5 \quad , \quad u/U \approx 7 \times 10^{-3} .$$

Therefore

$$R_{\ell_{120}}^{1/2} = \left(\frac{u\ell}{\nu}\right)^{1/2} = (3.5 \times 10^{-3})^{1/2}\left(\frac{Ud}{\nu}\right)^{1/2}$$
$$= (3.5 \times 10^{-3})^{1/2}\left(\frac{UL}{\nu}\right)^{1/2}\left(\frac{d}{L}\right)^{1/2}$$
$$\approx 1.3\, L^{1/2}$$

with L in cm. Thus, for the existence of an inertial range,

$$L^{1/2} >> 1,$$

and it appears that L = 100 cm will be adequate. Therefore there will be an inertial range at collapse if $L \gtrsim$ 1 meter.

Next, under what conditions will a buoyancy range exist? It is clear that, just as in considering R_ℓ, the crucial conditions occur at $x = 120d$. The buoyancy wavenumber K_b which separates the buoyancy range and the

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* Naudascher uses D for the diameter of the turbulence generator and L for the integral scale; our d and ℓ, respectively.

equilibrium range is (Lumley, 1964; Phillips, 1966)

$$K_b \approx (N^3/\epsilon)^{1/2}$$

where ϵ is the rate of energy dissipation. For a buoyancy range to exist

$$(1/\eta) >> K_b,$$

where

$$\eta = (\nu^3/\epsilon)^{1/4}$$

is the Kolmogorov micro-scale. Taking

$$\epsilon = u^3/\ell,$$

this condition is

$$u^3/(\nu \ell N^2) >> 1.$$

This can be written as

$$\left(\frac{u}{U}\right)^3\left(\frac{d}{\ell}\right)\left(\frac{D}{d}\right)\left(\frac{D}{L}\right)\left(\frac{UL}{\nu}\right)F^2 >> 1,$$

or with $u/U = 7 \times 10^{-3}$, $\ell/d = 0.5$ from Naudascher's results, and $U = 1$ m/sec.

$$L >> 1$$

with L in cm. Again, it seems reasonable that $L \gtrsim 1$ meter will be adequate.*

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* At $x = 120\ d$, the ratio u^3/ℓ is actually larger than ϵ by the amount of energy extracted in buoyancy range. Including this consideration, this condition is given by

$$L >>> 1.$$

Again, it seems reasonable that $L \approx 100$ cm is adequate.

Finally, it might be required that the Reynolds number based on the dissipation length scale, λ ,

$$R_\lambda = \left(\frac{u\lambda}{\nu} \right) >> 1.$$

Again consider only the case χ = 120 d . Naudascher (Figure 13) gives curves of $\lambda/\lambda_\infty (\lambda_\infty/d$ = 0.135) as a function of $\nu(\chi - \chi_0)/U\lambda_\infty^2$. Assuming that χ_0 = 0,

$$\left(\frac{\nu\chi}{U\lambda_\infty^2} \right) = \left(\frac{\nu}{Ud} \right)\left(\frac{\chi}{d} \right)\left(\frac{d}{\lambda_\infty} \right)^2$$

$$= \left(\frac{\nu}{UL} \right)\left(\frac{L}{d} \right)\left(\frac{\chi}{d} \right)\left(\frac{d}{\lambda_\infty} \right)^2 \approx 2/L$$

with L in cm. For $1m \leq L \leq 10m$, λ/λ_∞ lies in the range 0.5 to 0.2. Therefore it will be assumed that

$$\lambda/\lambda_\infty = 0.4 \qquad \text{and} \qquad \lambda_\infty/d = 0.135$$

to obtain an estimate of R_λ . Therefore,

$$R_\lambda = \left(\frac{u}{U} \right)\left(\frac{\lambda}{\lambda_\infty} \right)\left(\frac{\lambda_\infty}{d} \right)\left(\frac{d}{L} \right)\left(\frac{UL}{\nu} \right)$$

$$= 0.16\, L$$

with L in cm. Again, it appears that $L \approx$ 1 m will be satisfactory.

Conclusions

a) The ratio of the time for the wake to collapse to the time scale of the turbulence (t_c/t_T) will scale correctly with the Froude number $F = U/ND$ if the wake flow is turbulent at model scale.

b) For the model wake to be similar to that of the prototype, the requirements are:

for an inertial range,

$$R_\ell^{1/2} = \left[(u/U)(\ell/d)(d/L)\, R_L\right]^{1/2} \gg 1;$$

for a buoyancy range,

$$(u/U)^3 (d/\ell)(D/d)(D/L)\, F^2 R_L \gg 1;$$

and finally,

$$R_\lambda = (u/U)(\lambda/\lambda_\infty)(\lambda_\infty/d)(d/L) R_L \gg 1$$

with

$$R_L = UL/\nu .$$

Using Naudascher's results, in all cases these inequalities can be satisfied by taking R_L = 10^6, i. e., U = 1m/sec. L = 1 m with ν = 10^{-2}cm^2/sec.

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